Solving Quadratic Equations without Factoring

Recall: To solve by factoring,

(1) collect all terms on one side of equal sign
(2) factor the expression
(3) use (a)(b) = 0 to state a = 0 or b = 0

Consider this example:

M A N

$$x^2 - 12x + 32 = 0$$
$$(x-8)(x-4) = 0$$

$x - 8 = 0$ or $x - 4 = 0$

$x = 8$ $x = 4$

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L8(6.4)-Solving Quadratic Equations without Factoring

Vertex form can also be very useful for solving a quadratic equation.

Ex.1 (a) Write $y = x^2 - 12x + 32$ in vertex form
(b) Solve for y = 0

$y = x^2 - 12x + 32$
$= x^2 - 12x + 36 - 36 + 32$
$= (x-6)^2 - 36 + 32$
$y = (x-6)^2 - 4$

b) $y = 0$
$0 = (x-6)^2 - 4$
$\sqrt{4} = \sqrt{(x-6)^2}$
$\pm 2 = x - 6$
$6 \pm 2 = x$
$x = 6 \pm 2$
or
$x = 4$ $x = 8$

$b^2 - 4ac > 0$
$b^2 - 4ac = 0$
$b^2 - 4ac < 0$

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In some cases, one may be simpler than the other.

Ex.2 Write in factored & vertex form, then choose which to use for solving.

(a) $x^2 + 3x - 4 = 0$

M A N
-4 3 4 -1

$(x+4)(x-1)$

$x=-4$ or $x=1$

$0 = x^2+3x-4$

$= x^2+3x+2.25-2.25-4$

$= (x+1.5)^2-6.25$

$0 = (x+1.5)^2-6.25$

$\sqrt{6.25} = \sqrt{(x+1.5)^2}$

$\pm 2.5 = x+1.5$

$-1.5 \pm 2.5 = x$

$x=-4$ or $x=1$

(b) $x^2 - 9 = 7$

$x^2-9-7=0$

$x^2-16=0$

$=(x-4)(x+4)$

$x=4$ or $x=-4$

x^2-16 Vertex $(0,-16)$

$y=(x-0)^2-16$

$0 = x^2-16$

$\sqrt{16} = \sqrt{x^2}$

$\pm 4 = x$

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Ex.2 Write in factored & vertex form, then choose which to use for solving.

(b) $x^2 - 9 = 7$

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If factoring is not possible use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$ax^2 + bx + c$

which is derived from completing the square (p.337-338).

Note: To use the quadratic formula, the equation must be in standard form, $ax^2 + bx + c = 0$.

The '$\pm$' symbol means there are two solutions.

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

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Ex.3 Solve using the quadratic formula.

a) $x^2 - 4x - 3 = 0$ ($a = 1$, $b = -4$, $c = -3$)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 12}}{2}$$

$$= \frac{4 \pm \sqrt{28}}{2}$$

exact $\frac{4 + \sqrt{28}}{2}$ $\frac{4 - \sqrt{28}}{2}$:

b) $x^2 - 2x - 5 = 0$

$x_1 \doteq 3.45$ $x_2 \doteq -1.45$ $x_1 \doteq 4.65$ or $x_2 \doteq -0.65$

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Assigned Work:

p.343 # 1ad, 3, 4bdf, 5ace, 9ad, 10d, 14, 19*